B.K. BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS
A CBSE DAY-CUM-BOYS' RESIDENTIAL SCHOOL

PRE BOARD 1 (2025-26)
MATHEMATICS
MARKING KEY

Class: XII | **Time: 3hrs**
Date: 07/11/25 | **Max Marks: 80**
Admission no: | **Roll no:**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case-based integrated units of assessment (04 marks each) with sub-parts.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks have been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

	SECTION A				
1.	In what ratio must tea at ₹60/kg be mixed with tea at ₹80/kg so that the mixture costs ₹70/kg?				1m
	(a)1:1	(b) 2:3	(c) 3:2	(d) none of these	
2.	If $x \equiv 2(\text{mod}7)$, then $x^2 \equiv ?\ (\text{mod}7)$. (a) 2 (b) 4 (c) 1 (d) 3				1m
	(a) 4	(b) 2	(c) 1	(d) none of these	
3.	If $-2x > 6$, then x is:				1m
	(a) $x > -3$	(b) $x < -3$	(c) $x = -3$	(d) none of these	
4.	The order of matrix $A = [a_{ij}]$where $a_{ij} = 3i - 2j$ for $1 \le i \le 2, 1 \le j \le 3$is				1m
	(a) 2 × 2	*(b) 2 × 3*	(c) 3 × 2	(d) none of these	
5.	The sum of a matrix and its transpose is always				1m
	(a) Symmetric matrix	(b) Skew-symmetric matrix	(c) Diagonal matrix	(d) none of these	
6.	The determinant of a scalar multiple of a matrix kA (where $A$is an $n \times n$matrix) is				1m
	(a) k det(A)	*(b)* k^n *det*(A)	(c) n k det(A)	(d) none of these	
7	If $A$is a square matrix and $\lvert A \rvert = 0$, then A is				1m
	(a) Invertible	*(b) Singular*	(c) Orthogonal	(d) none of these	
8	If A is a 3 × 3matrix and det$(A) = 5$, then det (A^{-1}) =?				1m
	(a) 5	*(b) 1/5*	(c) –5	(d) none of these	
9	If $y = \log x$, then $\frac{dy}{dx}$ =?				1m
	(a) $\frac{1}{x}$	(b) x	(c) $\log x$	(d) none of these	

10	**If $\frac{d}{dx}F(x) = 6x^2$, then $F(x)$ =?**				**1m**
	(a) $2x^3 + C$	**(b)** $3x^2 + C$	**(c)** $x^3 + C$	**(d) none of these**	
11	**The order of the differential equation** $$\frac{d^2y}{dx^2} + (\frac{dy}{dx})^3 + y = 0$$ **is:**				**1m**
	(a) 2	**(b) 3**	**(c) 4**	**(d) none of these**	
12	**For a binomial distribution $B(n, p)$, the mean is**				**1m**
	(a)$np(1-p)$	**(b)**np^2	***(c)***np	**(d) none of these**	
13	**Poisson distribution is a limiting case of binomial distribution when**				**1m**
	(a)n is small, p is large	***(a)n is large, p is small***	**(c)n is small, p is small**	**(d) none of these**	
14	**The feasible region of an LPP is always:**				**1m**
	(a) A straight line	***(b) A convex polygon or unbounded region***	**(c) A circle**	**(d) none of these**	
15	**A subset of a population selected for analysis is called:**				**1m**
	(a) Parameter	***(b) Sample***	**(c) Variable**	**(d) none of these**	
16	**The standard deviation of the sampling distribution of the sample mean is called:**				**1m**
	(a) Standard deviation	***(b) Standard error***	**(c) Variance**	**(d) none of these**	
17	**The components of time series are:**				**1m**
	(a)Trend, seasonal, cyclical, irregular	**(b) Mean, median, mode, range**	**(c) Profit, loss, cost, revenue**	**(d) none of these**	
18	**If a perpetuity pays ₹2,000 every year and the rate of interest is 8% p.a., its present value is:**				**1m**
	(a) ₹16,000	**(b) ₹20,000**	***(c) ₹25,000***	**(d) none of these**	
19	**Assertion (A): The present value of a perpetuity decreases when the rate of interest increases.** **Reason (R): Present value of a perpetuity is inversely proportional to the rate of interest.**				**1m**
	(a) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A). **(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).** **(c) Assertion (A) is true and Reason (R) is false.** **(d) Assertion (A) is false and Reason (R) is true.**				
20	**Assertion (A): In a sinking fund, the total amount accumulated at the end of the period depends on the rate of interest and time.** **Reason (R): Sinking fund amount is calculated using the formula $A = R \times \frac{(1+i)^n - 1}{i}$.**				**1m**
	(a) Both Assertion (A) and Reason (R) are the true and Reason (R) is a correct explanation of Assertion (A). **(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).** **(c) Assertion (A) is true and Reason (R) is false.** **(d) Assertion (A) is false and Reason (R) is true.**				
	SECTION B				
21	**A boat goes 12 km downstream in 3 hours and 18 km upstream in 6 hours. Find the speed of the boat in still water.**				**2m**

A:-	Downstream speed = $12/3 = 4$ km/h. Upstream speed = $18/6 = 3$ km/h. Boat speed b = $\frac{\text{downstream} + \text{upstream}}{2} = \frac{4+3}{2} = \frac{7}{2} = 3.5$ km/h. Stream speed s = $\frac{4-3}{2} = 0.5$ km/h. Check: downstream $b + s = 3.5 + 0.5 = 4$ (matches), upstream $b - s = 3.5 - 0.5 = 3$ (matches).	**1m** **1m**
	Or	
	In what ratio must coffee costing ₹200 per kg be mixed with coffee costing ₹280 per kg so that the mixture may be worth ₹240 per kg?	
	Let cheaper = 200, dearer = 280, mean = 240 $$\text{Ratio} = (280 - 240) : (240 - 200) = 40 : 40 = 1 : 1$$ ✅ Required ratio = **1 : 1**	**1m** **1m**
22	**Four bad eggs are mixed with ten good ones. If three eggs are drawn one after another without replacement, find the probability distribution of the number of bad eggs drawn.**	**2m**
	Or	
	A fair die is thrown once. Let *X* denote the number obtained. Find the mean and variance of *X*.	
A:-	Total eggs = 14 (4 bad, 10 good). Draw 3 without replacement. Let X = number of bad eggs drawn. Then $X \in \{0, 1, 2, 3\}$ and $$P(X = k) = \frac{\binom{4}{k}\binom{10}{3-k}}{\binom{14}{3}}, \qquad k = 0, 1, 2, 3.$$ Since $\binom{14}{3} = 364$, compute: • $P(X = 0) = \frac{\binom{4}{0}\binom{10}{3}}{364} = \frac{1 \cdot 120}{364} = \frac{30}{91} \approx 0.3297.$ • $P(X = 1) = \frac{\binom{4}{1}\binom{10}{2}}{364} = \frac{4 \cdot 45}{364} = \frac{45}{91} \approx 0.4945.$ • $P(X = 2) = \frac{\binom{4}{2}\binom{10}{1}}{364} = \frac{6 \cdot 10}{364} = \frac{15}{91} \approx 0.1648.$ • $P(X = 3) = \frac{\binom{4}{3}\binom{10}{0}}{364} = \frac{4 \cdot 1}{364} = \frac{1}{91} \approx 0.0110.$ Check: $30/91 + 45/91 + 15/91 + 1/91 = 91/91 = 1.$ So the probability distribution is: k | 0 | 1 | 2 | 3 $P(X = k)$ | $30/91$ | $45/91$ | $15/91$ | $1/91$	**1m** **1m**
	Answer: Possible values: 1, 2, 3, 4, 5, 6 $$E(X) = \frac{1+2+3+4+5+6}{6} = 3.5$$ $$E(X^2) = \frac{1^2+2^2+3^2+4^2+5^2+6^2}{6} = \frac{91}{6}$$ $$Var(X) = E(X^2) - [E(X)]^2 = \frac{91}{6} - (3.5)^2 = 2.92$$ ✅ Mean = 3.5, Variance = 2.92	**1m** **1m**
23	**The probability of a man hitting a target in a single shot is $\frac{1}{4}$. How many times must he fire so that the probability of hitting the target at least once is greater than $\frac{2}{3}$?**	**2m**

A:-	Let the man fire n times. Probability of hitting in one shot $= p = \frac{1}{4}$. Probability of missing in one shot $= 1 - p = \frac{3}{4}$. $$P(\text{hitting at least once}) = 1 - P(\text{missing all})$$ $$\Rightarrow 1 - \left(\frac{3}{4}\right)^n > \frac{2}{3}$$ $$\Rightarrow \left(\frac{3}{4}\right)^n < \frac{1}{3}$$ Taking logarithms on both sides: $$n \log\left(\frac{3}{4}\right) < \log\left(\frac{1}{3}\right)$$ $$n > \frac{\log(1/3)}{\log(3/4)} = \frac{-\log 3}{\log(4/3)} \approx 3.82$$ $$\therefore n = 4$$	**1m** **1m**
24	**A machine is purchased for ₹30,000. Its scrap value after 13 years is estimated to be ₹4,000. Find the annual depreciation using the straight-line method.**	**2m**
A:-	$$\text{Annual Depreciation} = \frac{\text{Cost} - \text{Scrap Value}}{\text{Life}}$$ $$= \frac{30000 - 4000}{13} = \frac{26000}{13} = ₹2000$$	**1m** **1m**
25	**A start-up company invested ₹3,00,000 in shares for 5 years. The value of this investment was ₹3,50,000 at the end of the second year, ₹3,80,000 at the end of the third year, and on maturity, the final value stood at ₹4,50,000. Calculate the Compound Annual Growth Rate (CAGR) on the investment.** **[Given that $(1.5)^5 = 1.084$]**	**2m**
A:-	Formula for CAGR: $$\text{CAGR} = \left(\frac{\text{Final Value}}{\text{Initial Value}}\right)^{\frac{1}{n}} - 1$$ Substitute values: $$\text{CAGR} = \left(\frac{450000}{300000}\right)^{\frac{1}{5}} - 1$$ $$= (1.5)^{\frac{1}{5}} - 1$$ $$= 1.084 - 1 = 0.084$$ $$\text{CAGR} = 8.4\%$$ ✅ Compound Annual Growth Rate = 8.4%	**1m** **1m**
	SECTION C	
26	**Two pipes A and B can fill a cistern in 12 hours and 15 hours, respectively. A third pipe C can empty the cistern in 20 hours.** **All three pipes are opened together.** **(a) Find the time in which the cistern will be filled.** **(b) If pipe C is closed after 4 hours, find how much more time will be required to fill the cistern.**	**3m**

A:-

Rates (cistern/hour):

A = $1/12 = 5/60$

B = $1/15 = 4/60$

C (emptying) = $-1/20 = -3/60$

All three open: net rate

$$\frac{5+4-3}{60} = \frac{6}{60} = \frac{1}{10} \text{ cistern/hour}$$

1m

So time to fill = reciprocal = **10 hours**. ✅

If C is closed after 4 hours:

Work done in 4 hours = $4 \times \frac{1}{10} = \frac{4}{10} = \frac{2}{5}$.

Remaining = $1 - \frac{2}{5} = \frac{3}{5}$.

1m

A + B rate = $\frac{1}{12} + \frac{1}{15} = \frac{9}{60} = \frac{3}{20}$ cistern/hour.

Time to finish = $\frac{3/5}{3/20} = \frac{3}{5} \times \frac{20}{3} = 4$ hours.

1m

27	**In a 200 m race, A can run 200 m in 25 seconds, and B in 28 seconds.** **(a) By what distance does A beat B?** **(b) By what time does A beat B?**				**3m**

A:-

(a)

Speed of A = $\frac{200}{25} = 8$ m/s

Speed of B = $\frac{200}{28} = \frac{50}{7} \approx 7.14$ m/s

When A finishes 200 m in 25 s,

Distance covered by B in 25 s = $7.14 \times 25 = 178.5$ m

1m

So A beats B by $200 - 178.5 = 21.5$ m

✅ A beats B by **21.5 m**

(b)

Alternatively, time taken by B to run 200 m = 28 s

Time taken by A = 25 s

So, A beats B by $28 - 25 = 3$ seconds

✅ A beats B by **3 seconds**

2m

28	**Show that the local maximum value of x + 1/x is less than local minimum value.**	**3m**

A:-

1. Differentiate:

$$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}.$$

Critical points when $f'(x) = 0$ give $x^2 - 1 = 0$, so $x = \pm 1$.

2. Second derivative:

$$f''(x) = \frac{d}{dx}\left(1 - \frac{1}{x^2}\right) = \frac{2}{x^3}.$$

Evaluate at the critical points:

- $f''(1) = 2 > 0 \rightarrow x = 1$ is a **local minimum**. Value: $f(1) = 1 + 1 = 2$.
- $f''(-1) = \frac{2}{(-1)^3} = -2 < 0 \rightarrow x = -1$ is a **local maximum**. Value: $f(-1) = -1 - 1 = -2$.

2m

Thus the local maximum value is -2 and the local minimum value is 2. Clearly

$$\boxed{-2 < 2},$$

so the local maximum value is less than the local minimum value.

1m

Or

	Show that the local maximum value of $f(x) = x^2 + \frac{1}{x^2}$ is greater than its local minimum value.	
A:-	Solution: $$f(x) = x^2 + \frac{1}{x^2}$$ Differentiate: $$f'(x) = 2x - \frac{2}{x^3} = \frac{2(x^4 - 1)}{x^3}$$	**1m** **2m**
29	**Solve the differential equation:** $$\frac{dy}{dx} = e^{x+y} + x^2 e^y$$	**3m**
	Or **Solve the differential equation** $$\frac{dy}{dx} = e^{x+y} + xe^y$$	
A:-	We have $$\frac{dy}{dx} = e^{x+y} + x^2 e^y = e^y(e^x + x^2).$$ This is separable. Divide by e^y and separate variables: $$e^{-y}\,dy = (e^x + x^2)\,dx.$$ Integrate both sides: $$\int e^{-y}\,dy = \int (e^x + x^2)\,dx \quad \Rightarrow \quad -e^{-y} = e^x + \frac{x^3}{3} + C,$$	**1m** **1m** **1m**
	Factor out e^y: $$\frac{dy}{dx} = e^y(e^x + x)$$ Separate variables: $$e^{-y}dy = (e^x + x)dx$$ Integrate both sides: $$\int e^{-y}dy = \int (e^x + x)dx$$ $$-e^{-y} = e^x + \frac{x^2}{2} + C$$ Simplify: $$\boxed{e^{-y} + e^x + \frac{x^2}{2} = C}$$	**1m** **2m**
30	**A company produces two types of biscuits, A and B.** **Each packet of A requires 4 kg of flour and 2 kg of sugar.** **Each packet of B requires 2 kg of flour and 4 kg of sugar.** **The company has 40 kg of flour and 48 kg of sugar available per day.**	**3m**

	If the profit per packet of A and B is ₹6 and ₹8 respectively, formulate an LPP to maximize profit.	
A:-	Let x, y be packets of A and B. Maximize $Z = 6x + 8y$ subject to $$2x + y \leq 20, \qquad x + 2y \leq 24, \qquad x, y \geq 0.$$	**2m** **1m**
	Or	
31	**A company establishes a sinking fund to repay a debt of ₹2,50,000 due in 4 years. Annual contributions are made at the end of each year. Find the amount of each annual deposit if the rate of interest is 18% per annum compounded annually.**	**3m**
A:-	Given, $$A = ₹2,50,000, \quad i = 18\% = 0.18, \quad n = 4$$ The formula for a sinking fund is: $$A = R \times \frac{(1+i)^n - 1}{i}$$ Substitute the given values: $$2,50,000 = R \times \frac{(1+0.18)^4 - 1}{0.18}$$ Now calculate step-by-step: $$(1+0.18)^4 = (1.18)^4 = 1.938$$ $$(1.938 - 1) = 0.938$$ $$\frac{0.938}{0.18} = 5.211$$ So, $$2,50,000 = R \times 5.211$$ $$R = \frac{2,50,000}{5.211} = 47,990.78$$	**1m** **1m** **1m**
	SECTION D	
32	**Solve by matrix method:** $$2x + y + z = 10, 3x + 2y + 3z = 18, x + 4y + 2z = 12$$	**5m**

A:-	Solution outline: $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 10 \\ 18 \\ 12 \end{bmatrix}$ $\lvert A\rvert = 2(2\cdot 2 - 3\cdot 4) - 1(3\cdot 2 - 3\cdot 1) + 1(3\cdot 4 - 2\cdot 1) = 2(4-12) - (6-3) + (12-2) = -16 - 3 + 10 = -9.$ Non-zero ⇒ unique solution. After solving: $x = 2,\ y = 3,\ z = 2.$ ✅	**2m** **2m** **1m**
33	**The fixed cost of producing an item is ₹6500 and the variable cost per item is ₹12.50. Find:** **(i) The cost function.** **(ii) The total cost of producing 75 items.** **(iii) The average cost of producing 400 items.** **Also, draw a suitable graph showing Fixed Cost (TFC), Variable Cost (TVC) and Total Cost (TC).** **Or** **A firm has a fixed cost of ₹10,000 and the variable cost per item is ₹18.** **Find:** **(i) The cost function** **(ii) The total cost of producing 150 items** **(iii) The average cost of producing 400 items**	**5m**
A:-	Given: Fixed Cost (TFC) = ₹6500 Variable Cost per unit = ₹12.50 (i) Cost Function $C(x) = 6500 + 12.5x$ where x = number of items produced. (ii) Total Cost for 75 items $C(75) = 6500 + 12.5(75) = 6500 + 937.5 = ₹7437.50$ (iii) Average Cost for 400 items $\text{Average Cost} = \frac{C(400)}{400} = \frac{6500 + 12.5(400)}{400}$ $= \frac{6500 + 5000}{400} = \frac{11500}{400} = ₹28.75$	**1m** **2m** **2m**

$$C(x) = 10000 + 18x$$

(ii) $C(150) = 10000 + 18(150) = 10000 + 2700 = 12700$

(iii) $AC(400) = \frac{10000+18(400)}{400} = \frac{10000+7200}{400} = 43$ **2m**

✅ Answers:

(i) $C(x) = 10000 + 18x$ **1m**

(ii) ₹12,700

(iii) ₹43 **2m**

34 **The production of cement (in tonnes) by a firm during 9 years is given below:** **5m**

Year	1	2	3	4	5	6	7	8	9
Production(tonnes)	4	5	5	6	7	8	9	8	10

Calculate the trend values for the above series using the 3-yearly moving average method.

A:-

Trend Values Summary:

Year	Trend Value
2	4.67
3	5.33
4	6.00
5	7.00
6	8.00
7	8.33
8	9.00

2m

2m

✅ Final Trend Values (3-yearly moving averages):

4.67, 5.33, 6.00, 7.00, 8.00, 8.33, 9.00 **1m**

35 **Madhu exchanged her old car valued at ₹1,50,000 for a new car priced at ₹6,50,000. She paid ₹x as down payment and agreed to pay the balance in 20 equal monthly instalments of ₹21,000 each. The rate of interest charged is 9% per annum, compounded monthly. Find the value of x, given that $(1.0075)^{-20} = 0.86118985$.** **5m**

Or

Rohit exchanged his old bike valued at ₹40,000 for a new one priced at ₹1,60,000. He paid ₹x as down payment and agreed to pay the balance in 12 equal monthly instalments of ₹10,000 each. Interest rate = 10% p.a., compounded monthly. Given: $(1.008333)^{-12} = 0.9052868$ Find the value of x.

A:-	Total due after exchange = ₹6,50,000 − ₹1,50,000 = ₹5,00,000. Present value of 20 monthly instalments $R = 21,000,\ r = 0.0075,\ n = 20$: $$P = R \cdot \frac{1-(1+r)^{-n}}{r} = 21000 \cdot \frac{1-0.86118985}{0.0075} = 21000 \times 18.50802 = ₹3,88,668.42$$ Down payment $x = 5,00,000 - 3,88,668.42 = ₹1,11,331.58 \approx ₹1,11,332.$ Answer: **₹1,11,332 (approx.)**	**1m** **2m** **2m**
	Price difference = ₹(1,60,000 − 40,000) = ₹1,20,000 Loan amount = ₹(1,20,000 − x) $$(1,20,000 - x) = 10,000 \times \frac{1-(1.008333)^{-12}}{0.008333}$$ $$= 10,000 \times \frac{1-0.9052868}{0.008333} = 10,000 \times \frac{0.0947132}{0.008333} = 10,000 \times 11.3656 = 1,13,656$$ $$1,20,000 - x = 1,13,656 \Rightarrow x = 6,344$$ ✅ Answer: ₹6,344	**1m** **2m** **2m**
	SECTION E	
36	**A digital safe uses a 4-digit PIN where each digit follows modulo 10 arithmetic (i.e., after 9, it goes back to 0).** **For example, adding 7 to 6 gives $(7+6)(\mathbf{mod10}) = 3$.** **The original PIN is 5836. The safe's system performs the following operations each time it's accessed:** • **Add 7 to the first digit,** • **Add 8 to the second digit,** • **Add 9 to the third digit,** • **Add 6 to the fourth digit, all under modulo 10.** **Answer the following:** **(i) Find the new PIN after the first access.** **(ii) Find the PIN after the second access (apply the same operation again).** **(iii) How many accesses will return the PIN to its original form?** **Or** **(iii) If the third digit after 5 accesses becomes 5, verify the result using modular arithmetic.**	**4m**
A:-	(a) **2622** (b) **9418** (c) **10** accesses (d) After 5 accesses the third digit = **8** (so it does **not** become 5).	**1m** **1m** **2m**
37	**The total profit function of a company is given by** $$\boldsymbol{P(x) = -5x^2 + 125x + 37500}$$ **where x represents the number of units produced.** **Based on the above information, answer the following:** **(i) What will be the production when the profit is maximum?** **(ii) What will be the maximum profit?** **Or** **When the production is 2 units, what will be the profit of the company?** **(iii) Find the interval in which the profit function is strictly increasing.**	**4m**

A:-	(i) Production for maximum profit = **12.5 units** (ii) Maximum profit = **₹38,281.25** **Or** Profit when 2 units are produced = **₹37,730** (iii) Profit is strictly increasing for **x < 12.5**	**1m** **2m** **1m**
38	**An urn contains 25 balls, 10 marked *X* and 15 marked *Y*. A ball is drawn at random, its mark noted, and then it is replaced. Six balls are drawn in this way. Find:** **(i) Probability that all six bear mark *X*.** **(ii) Probability that not more than 2 balls bear mark *Y*.** **(iii) Probability that the number of balls with *X* and *Y* marks are equal.** **Or** **(iii) Probability that at least one ball bears mark *Y*.**	**4m**
A:-	(i) $P(\text{all 6 are } X) = \left(\frac{2}{5}\right)^6 = \frac{64}{15625} = 0.004096.$ (ii) $P(\text{not more than 2 are } Y) = P(Y \leq 2) = \frac{112}{625} = 0.1792.$ (iii) $P(\text{equal numbers of } X \text{ and } Y) = P(X = 3) = \frac{864}{3125} = 0.27648.$ Or (alternate iii) $P(\text{at least one } Y) = 1 - \left(\frac{2}{5}\right)^6 = \frac{15561}{15625} = 0.995904.$	**1m** **1m** **2m**

*****************Best of luck****************